

Chapter 23

Interface of Large Scale and Micro-physics

23.1 Structure in the Universe

23.2 The Inflationary Universe

The dynamical equations are

$$\frac{\ddot{R}}{R} = -\frac{1}{6}(\rho + 3p) \quad (23.1)$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{3}\rho - \frac{k}{R^2} \quad (23.2)$$

with k as usual the sign of the curvature.

Fill the space with a scalar field,

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu} (\partial_\mu\phi) (\partial_\nu\phi) - V(\phi). \quad (23.3)$$

The Euler-Lagrange equation is

$$\square^2\phi + \frac{dV}{d\phi} = 0, \quad (23.4)$$

and the energy momentum tensor is

$$T_{\mu\nu} = (\partial_\mu\phi) (\partial_\nu\phi) - g_{\mu\nu} \left\{ \frac{1}{2} (\partial_\sigma\phi) (\partial^\sigma\phi) - V(\phi) \right\}. \quad (23.5)$$

Comparing this to a Pascal fluid,

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \quad (23.6)$$

where u_μ is the fluid four velocity field, the $T_{\mu\nu}$ in a local rest frame is

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \quad (23.7)$$

where the ρ and p have the usual interpretation of energy density and pressure respectively. Thus the energy density and pressure of the scalar field are

$$\begin{aligned} \rho_\phi &= \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2} (\vec{\nabla}\phi)^2 \\ p_\phi &= \frac{1}{2} \dot{\phi}^2 - V(\phi) - \frac{1}{2} (\vec{\nabla}\phi)^2. \end{aligned} \quad (23.8)$$

In a spatially homogeneous universe, $\vec{\nabla}\phi \approx 0$ and, if it is also temporally slowly varying,

$$\rho_\phi = V(\phi) = -p. \quad (23.9)$$

The scalar field produces an effect that is the same as that of a cosmological constant term in the Einstein equation, Equation ???. This same observation can be seen directly from a comparison of Equation 23.7 with $\lambda g_{\mu\nu}$

It is consistent with the cosmological principal to use spatial homogeneity to require that $\phi(t)$ only. Thus the scalar field is dynamically the same as a point particle with a potential $V(\phi)$.

The equations of motion for ϕ follow from the hydrodynamics of the Pascal fluid,

$$\partial_\mu T^{\mu\nu} = 0. \quad (23.10)$$

This reduces to

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad (23.11)$$

where H is the FRW Hubble constant $\frac{\dot{R}}{R}$. This is the classical mechanics problem of a point particle sliding down a potential hill with the “friction” set by H .

Assuming that at some time t , ρ is dominated by ϕ

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{1}{3}\rho_\phi = \frac{1}{3} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right] \quad (23.12)$$

and thus we know H

To get $\ddot{R} > 0$, we need $\dot{\phi} < V(\phi)$, see Equation 23.8. Inflation in the “slow roll” approximation

23.3 String Theory

23.4 Quantum Gravity

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